General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MFP2**
Unit Further Pure 2

Thursday 15 May 2008 9.00 am to 10.30 am

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Express

$$5\sinh x + \cosh x$$

in the form $A\mathrm{e}^{x} + B\mathrm{e}^{-x}$, where A and B are integers. *(2 marks)*

(b) Solve the equation

$$5\sinh x + \cosh x + 5 = 0$$

giving your answer in the form $\ln a$, where a is a rational number. *(4 marks)*

2 (a) Given that

$$\frac{1}{r(r+1)(r+2)} = \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$$

show that $A = \frac{1}{2}$ and find the value of B. *(3 marks)*

(b) Use the method of differences to find

$$\sum_{r=10}^{98} \frac{1}{r(r+1)(r+2)}$$

giving your answer as a rational number. *(4 marks)*

3 The cubic equation

$$z^3 + qz + (18 - 12\mathrm{i}) = 0$$

where q is a complex number, has roots α, β and γ.

(a) Write down the value of:

(i) $\alpha\beta\gamma$; *(1 mark)*

(ii) $\alpha + \beta + \gamma$. *(1 mark)*

(b) Given that $\beta + \gamma = 2$, find the value of:

(i) α; *(1 mark)*

(ii) $\beta\gamma$; *(2 marks)*

(iii) q. *(3 marks)*

(c) Given that β is of the form $k\mathrm{i}$, where k is real, find β and γ. *(4 marks)*

4 (a) A circle C in the Argand diagram has equation

$$|z + 5 - \mathrm{i}| = \sqrt{2}$$

Write down its radius and the complex number representing its centre. *(2 marks)*

(b) A half-line L in the Argand diagram has equation

$$\arg(z + 2\mathrm{i}) = \frac{3\pi}{4}$$

Show that $z_1 = -4 + 2\mathrm{i}$ lies on L. *(2 marks)*

(c) (i) Show that $z_1 = -4 + 2\mathrm{i}$ also lies on C. *(1 mark)*

(ii) Hence show that L touches C. *(3 marks)*

(iii) Sketch L and C on one Argand diagram. *(2 marks)*

(d) The complex number z_2 lies on C and is such that $\arg(z_2 + 2\mathrm{i})$ has as great a value as possible.

Indicate the position of z_2 on your sketch. *(2 marks)*

Turn over ▶

5 (a) Use the definition $\cosh x = \frac{1}{2}(e^x + e^{-x})$ to show that $\cosh 2x = 2\cosh^2 x - 1$. *(2 marks)*

(b) (i) The arc of the curve $y = \cosh x$ between $x = 0$ and $x = \ln a$ is rotated through 2π radians about the x-axis. Show that S, the surface area generated, is given by

$$S = 2\pi \int_0^{\ln a} \cosh^2 x \, dx$$ *(3 marks)*

(ii) Hence show that

$$S = \pi\left(\ln a + \frac{a^4 - 1}{4a^2}\right)$$ *(5 marks)*

6 By using the substitution $u = x - 2$, or otherwise, find the exact value of

$$\int_{-1}^{5} \frac{dx}{\sqrt{32 + 4x - x^2}}$$ *(5 marks)*

7 (a) Explain why $n(n+1)$ is a multiple of 2 when n is an integer. *(1 mark)*

(b) (i) Given that

$$f(n) = n(n^2 + 5)$$

show that $f(k+1) - f(k)$, where k is a positive integer, is a multiple of 6. *(4 marks)*

(ii) Prove by induction that $f(n)$ is a multiple of 6 for all integers $n \geqslant 1$. *(4 marks)*

8 (a) (i) Expand

$$\left(z+\frac{1}{z}\right)\left(z-\frac{1}{z}\right) \qquad \textit{(1 mark)}$$

(ii) Hence, or otherwise, expand

$$\left(z+\frac{1}{z}\right)^4\left(z-\frac{1}{z}\right)^2 \qquad \textit{(3 marks)}$$

(b) (i) Use De Moivre's theorem to show that if $z = \cos\theta + \mathrm{i}\sin\theta$ then

$$z^n + \frac{1}{z^n} = 2\cos n\theta \qquad \textit{(3 marks)}$$

(ii) Write down a corresponding result for $z^n - \dfrac{1}{z^n}$. *(1 mark)*

(c) Hence express $\cos^4\theta\sin^2\theta$ in the form

$$A\cos 6\theta + B\cos 4\theta + C\cos 2\theta + D$$

where A, B, C and D are rational numbers. *(4 marks)*

(d) Find $\displaystyle\int \cos^4\theta\sin^2\theta \, \mathrm{d}\theta$. *(2 marks)*

END OF QUESTIONS

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